Math 270: Differential Equations Day 13 Part 1

<u>Section 4.6</u>: Variation of Parameters Part 2

Method of Variation of Parameters

To determine a particular solution to $ay'' + by' + cy = f$:

(a) Find two linearly independent solutions $\{y_1(t), y_2(t)\}$ to the corresponding homogeneous equation and take

$$y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t).$$

(b) Determine $v_1(t)$ and $v_2(t)$ by solving the system in (9) for $v_1'(t)$ and $v_2'(t)$ and integrating.

$$(9) \qquad \begin{aligned} y_1 v_1' + y_2 v_2' &= 0 \\ y_1' v_1' + y_2' v_2' &= \frac{f}{a} \end{aligned}$$

(c) Substitute $v_1(t)$ and $v_2(t)$ into the expression for $y_p(t)$ to obtain a particular solution.

Example 3   Find a particular solution of the variable coefficient linear equation $t^2 y'' - 4ty' + 6y = 4t^3$, $t > 0$, given that $y_1(t) = t^2$ and $y_2(t) = t^3$ are solutions to the corresponding homogeneous equation.